On The Invariant Measures for Stochastic Evolution Equations in Hilbert Space

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Abstract
We examined the invariant measures for stochastic evolution equations in Hilbert space and derive a sufficient condition for the existence of an invariant measure in a global case for a more corrupted noise than Wiener processes for a stochastic evolution equation on a separable Hilbert space H defined by $dX = (AX(t) + F(X(t))dt + B(X(t))dZ(t)\ ,\ t \geq 0\ , X(0) = \eta\ ,$
where $\eta \in H, A$ *is a linear operator ,* F *is a bounded mapping from* H *into* H, Z *takes values in a separable Hilbert space,* U *and* $B,$ *is a bounded mapping from* H *into space of linear continuous operators from* U *into* $H.$

Keywords*: Stochastic Evolution Equation, Hilbert space, Semi-linear stochastic evolution equation.*

1. INTRODUCTION:

The study on invariant measures for (stochastic) dynamical systems is an importanttopic in the theory of (stochastic) dynamical systems. These measures provide certain invariant characterization such as ergodicity, strong or weak asymptotic stability forthe processes described by the systems. The research on the existence, uniqueness and regularity of invariant measures for stochastic evolution equations in Hilbertspaces has received a lot of attention (see [4,5,6,11]). Some useful methods have been developed in dealing with the invariant measures of stochastic evolution equations in Hilbert spaces in terms of different conditions on the coefficients of the stochastic evolution equations (SEEs). For the existence of invariant measures, the Krylov–Bogoliubov criterion is a powerful tool. Some efficient methods (so-called compactness method and dissipativity method) have been used by Da Prato, Gatarek and Zabczyk [3] and were further applied to study some specific equations. There are mainly three ways to show the uniqueness of invariant measures. The first one is to verify the strong Feller property (SFP) and irreducibility (I) (see [1,2,10,12]). In this case, one usually had to assume that the noise term is nondegenerate. The second one is the so-called Lyapunov approach (see [7,8,9]). In this case, the SEE's admit degenerate noise. The third one is the dissipative method developed by Da Prato, G¸atarek and Zabczyk [3].In this case, the drift coefficients can satisfy some dissipativity (for example, havingcertain polynomial growth). There are also some other methods of proving uniqueness of invariant measures (see [11]). The regularity of invariant measures in the Hilbert space setting has been studied by Da Prato and Zabczyk.

In recent times, the growing interest in stochastic evolution equations with white noise driven by various dimensional Wiener process has received robust research. Extensive works of the existence and uniqueness have been studied see [13 and 15].The research of infinite dimensional equations driven by not well developed, especially in the area of invariant measure in the case of Gaussian

noise [15].

2. Invariant Measures

This paper extend the work on the existence of invariant measure in infinite dimensional Wiener process of a stochastic evolution equation in a separable Hilbert space driven by Hilbert space valued process.

Consider a semi-linear stochastic evolution equation (SEE) in a separable real Hilbert space$(U, \|\cdot\|_u)$.

We consider a stochastic evolution equation on a separable Hilbert space H defined by $dX = (AX(t) + F\big(X(t)\big)dt + B\big(X(t)\big)dZ(t)\ ,\ t \geq 0$

$X(0) = \eta\ ,$ (1)

Where the noise is driven by Hilbert space valued process $(Z(t))_{t\geq 0}$ with stationary and independent increments and locally bounded second moments.

If the semigroup, generated by the part of A is hyperbolic and Lipschitz constants of the nonlinearities $F\ and\ G$ are sufficiently small, then the existence of a bounded solution implies existence of an invariant measure.

Let $H\ and\ E$ be separable real Hilbert spaces, let A be the generator of a strongly continuous semigroup $(S(t))_{t\geq 0}\ in\ E, let\ F: E \to E\ and\ G: E \to L(H, E)$ be defined as $\|F(x) - F(y)\| \leq L_F\|x - y\|$,

$\|G(x) - G(y)\| \leq L_G\|x - y\|,\ \forall x, y \in E, for\ some\ constants\ L_F, L_G > 0.$

$Let\ L(H, E)$denotes the space of all bounded linear operators from $H\ to\ F\ and$ $\|\cdot\|$ be the norm. Let Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space and $(\mathcal{F}_t)_{t\geq 0}$ be a filtration in $\mathcal{F}$, let $(Z(t))_{t\geq 0}$ be a family of random variables on $(\Omega, \mathcal{F}, \mathcal{P})$such that

$Z(t)\ is\ \mathcal{F}_t$measurable, $Z(t + u) - Z(t)$ is independent of $\mathcal{F}_t$.

$Z(t + u) - Z(t)\ and\ Z(s + u) - Z(s)$be of the same distribution $\forall\ s, t, u \geq 0$

such that $Z(0) = 0\ and\ \ sup_{0\leq t\leq T} E\|Z(t)\|^2 < \infty\ \ \forall\ T \geq 0.$

THEOREM1 : Assume condition in (1) and assume that there exist $M \geq 0\ and\ \alpha > 0$ such that

$\|S(t)\| \leq Me^{-\alpha t}\ \forall\ t \geq 0.\ Let\ \Lambda = ((\mathbb{E}\|Z(1) - \mathbb{E}Z(1)\|^2)^{1/2}\ if\ L_F\ and\ L_G\ are\ small\ that$

$6M^2(L_F^2/\alpha\ + L_G^2\Lambda) < \infty$, then there exist a unique invariant measure
$\mu\ of\ (1)\ with \int_E \|x\|^2 d\mu(x) < \infty$

Consider processes on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let $Z(t)$ be a Lévy process taking values in a separable Hilbert space $(U, \|\cdot\|_u)$.
Let $Z(t)$ be associated with two measures on U and Let the measure of jumps of Z be μ and Lévy measure of Zbe v.

Let $\mu([0, t], \Gamma) = \sum_{0\leq s\leq t\leq} 1_\Gamma\big(Z(s) - Z(s^-)\big),$
$tv(\Gamma) = \mathbb{E}(\mu([0, t], \Gamma))$ (2)

Where Γ is a Borel subset of U such that $\bar{\Gamma} \subseteq U \setminus \{0\}$.

It implies that $v(\{0\}) = 0$ and $\int_u \min(\|\cdot\|_u^2, 1)\, v(dy) < \infty$ (3)

The Lévy process $Z(t)$ is represented by

$$Z(t) = at + W(t) + \int_0^t \int_{\|y\|_{u\leq 1}} y(\mu(dy, ds) - v(dy)(ds) + \int_0^t \int_{\|y\|_{u>1}} y\mu(dy, ds) \quad (4)$$

where$a \in U, W$ is Wiener process with value points in U with covariance operator Q.

Let $L(H)$ be the space of linear continuous operator from another separable Hilbert space $(H, \|\cdot\|)$. Also Let $L(U, H)$ be the space of linear continuous operator from $U \; into \; H$.

Consider a semi-linear stochastic evolution equation in a separable real Hilbert space H written

as $dX(t) = [AX(t) + \mathcal{F}(X(t)]dt + G(X(t)dZ \;,\; t \geq 0,\; X(0) = \eta.$ (5)

where$\eta \in H$, the noise driven by Hilbert space valued process $(Z(t))_{t\geq 0}$ with stationary and independent increments and locally bounded second moments, A is a linear operator with dense domain, $\mathcal{F}$ is a bounded mapping from $H \; to \; L(U, H)$.

We state the following conditions:

(1) $C = \int_u \|y\|_u^2 v(dy) < \infty$

(2) A is the infinitesimal generator of a strongly continuous semi-group on H,

(3) $\exists\, \mathcal{L}_\alpha > 0$ such that $\|\alpha(x) - \alpha(y)\| \leq \mathcal{L}_\alpha \|x - y\|, \forall x, y \in H \; for \; some \;\; \mathcal{L}_\alpha > 0$

(4) $\exists \mathcal{L}_\beta > 0 \; such \; that \; \|\beta(x) - \beta(y)\|_{\mathcal{L}(U,H)} \leq \mathcal{L}_\beta \|x - y\|$

Now, condition (1) implies the existence of $\int_{\|y\|_u>1} yv(dy)$ u

We have, $\int_{\|y\|_u>1} \|y\|_u v(dy) \leq \int_{\|y\|_u>1} \|y\|_u^2 v(dy) \leq \int_u \|y\|_u^2 v(dy) < \infty$ (6)

$\exists \bar{\beta} = \int_{\|y\|_u>1} yv(dy) \in U$

Then $Z(t) = at + W(t) + \int_0^t \int_{\|y\|_{u\leq 1}} y(\mu(dy, ds) - v(dy)(ds) + \int_0^t \int_{\|y\|_{u>1}} y\mu(dy, ds)$

$$-\int_0^t \int_{\|y\|_u>1} yv(dy, ds) + bt \quad (7)$$

$$=(a + b)t + W(t) + \int_0^t \int_u y(\mu(dy, ds) - v(dy)ds) \quad (8)$$

So that, $EZ(1) = a + b \; and \; VarZ(1) = Var\, W(1) + \int_0^1 \int_u \|y\|^2 v(dy)ds = Tr\Lambda + k$ (9)

For process$Z(t), t \geq 0, Let\; \hat{Z}(t)$ be a process defined by

$$\hat{Z}(t) = \begin{cases} Z(t) \quad , & t \geq 0 \\ Z_L(-t)\,, & t < 0 \end{cases} \quad , t \in \mathbb{R} \tag{10}$$

where$(Z_L(t))$ is a Lévy process with same distribution as $(Z \quad (t))$ and independent of

$(Z \quad (t)) \quad _{t\geq 0}$.

DEFINITION 1:Let H be a separable real Hilbert space. Let a probability space be given as $(\Omega, \mathcal{F}, \mathbb{P})$. Let $Z = (Z(t))_{t\geq 0}$ be a family of H-valued random variables defined on $(\Omega, \mathcal{F}, \mathbb{P})$.

Then Z is a process with independent increments.

If (a) for every $t \geq s \geq 0$ the increment $Z(t) - Z(s)$is independent of the σ $-$algebragenerated by$\{Z(u): 0 \leq u \leq s\}, Z$ has stationary increment,

If (b) for every $s, t, u \geq 0$, the increments $\big(Z(t+u) - Z(u)\big)$ *and* $(Z(s+u) - Z(s))$ have the same distribution.

The family $(Z(t))$ is called a Lévy process if in addition to (a) and (b), we have

(c) $Z(0) = 0,\ \mathbb{P} - a.s.$

(d) $t \to Z(t)$ is continuous in probability; if $t \to t_0, then\ \mathbb{P}(\|Z(t) - Z(t_0)\| > \epsilon)\ \forall \epsilon > 0$.

The process Z is called $(\mathcal{F}_t)_t$ stationary independent increments process if Z is

$(\mathcal{F}_t)_t$adapted.

The Z has stationary increments, and for every $t \geq s \geq 0$ the increment $Z(t) - Z(s)$ is

independent of $(\mathcal{F}_s)$.

LEMMA 1:Let $(H, \|\cdot\|)$ be a normed space. Let $T > 0$ and Let$u: [0, T] \to H$ be

additive, that is, $u(s+t) = u(s) + u(t)\quad for\ every\ s, t \geq 0\ with\ s + t \leq T$. If u is

bounded on $[0, T]$, then $u(t) = \left({}^{t}/_{T}\right) u(T)\ \ \forall\, t \in [0, T]$. (11)

LEMMA2:If $sup_{0\leq t\leq T}\mathbb{E}\|Z(t)\| < \infty\ \ \forall\, T > 0\,, then\ \mathbb{E}Z(t) = t\mathbb{E}Z(1)\ \forall\, t \geq 0.$ (12)

If $\mathbb{E}\|Z(t)\|^2 < \infty\ \ \forall\, t > 0$, then $\mathbb{E}\left\|\hat{Z}(t)\right\|^2 = t\mathbb{E}\left\|\hat{Z}(1)\right\|^2\ \ \forall\, t \geq 0,$

where$\hat{Z}(t) = Z(t) - \mathbb{E}Z(t), t \geq 0.$ (13)

Proof: If $\mathbb{E}\|Z(t)\| < \infty$ then $Z(t)$ is Bochner integrable, implies that $\mathbb{E}Z(t) \in H$ is well defined. Given that $s, t \geq 0$ then:

$$\mathbb{E}Z(s+t) = \mathbb{E}[Z(s+t) - Z(t)] + \mathbb{E}Z(t) = \mathbb{E}[Z(s) - Z(0)] + \mathbb{E}Z(t) = \mathbb{E}Z(s) + \mathbb{E}Z(t)$$

Hence $t \to \mathbb{E}Z(t)$ is additive and $\|\mathbb{E}Z(t)\| \leq \mathbb{E}\|Z(t)\|$ bounded on $[0, T]$.

If $\mathbb{E}\|Z(t)\|^2 < \infty,\ then\ \mathbb{E}\|Z(t)\| \leq (\mathbb{E}\|Z(t)\|^2)^{1/2} < \infty$, and given that $\tilde{Z}(t)$ is well defined, also $\mathbb{E}\left\|\tilde{Z}(t)\right\|^2 < \infty\ \ \forall\, t \geq 0$. For $, t \geq 0$, then ,

$$\mathbb{E}\left\|\tilde{Z}(s+t)\right\|^2 = \mathbb{E}\left[\left\|\tilde{Z}(s+t)-\tilde{Z}(t)\right\|^2 + 2\langle \tilde{Z}(s+t)-\tilde{Z}(t), \tilde{Z}(t)\rangle + \left\|\tilde{Z}(t)\right\|^2\right]$$

$$\mathbb{E}\left\|\tilde{Z}(s)\right\|^2 + \mathbb{E}\left\|\tilde{Z}(t)\right\|^2 .$$

Corollary: If $v: [0,\infty) \to [0,\infty)$ is additive, that is $v(s+t) = v(s) + v(t)\ \ \forall\, s,t \geq 0$, then$v(t) = tv(1)\ \ \forall\, t \geq 0$.

Proof: Let $T > 0$ be arbitrary. The function v is additive on $[0,T]$. As $v(t) \geq 0\ \ \ \forall\, t$, V is monotonically increasing in the domain $[0,T]$. Hence $0 = v(0) \leq v(T)\ \ \forall\, t \in [0,T]$.

DEFINITION 2: Given a separable Hilbert space with inner product$\langle \cdot ,\cdot \rangle_H$ *and dual* H^*, *norm* $|\cdot|$, and a Markovsemigroup$\mathcal{P}_t$, $\mathcal{P}_t\Phi(x) = \int_H \Phi(y)\pi_t(x,dy),\ \ t \geq 0,\ \ \Phi \in \mathcal{C}_b(H)$ *on* H.

Let $\mathcal{P}_t$ be Feller, that is $\Phi \in \mathcal{C}_b(H) \Rightarrow \mathcal{P}_t\Phi \in \mathcal{C}_b(H)\ \ \forall\, t \geq 0$.

On the existence of invariant measure, we shall prove the Krylov-Bogoliubov theorem with concrete example.

Assume that for some$x_0 \in H$ $,\exists$ *the* $\lim_{t\to\infty} \mathcal{P}_t\Phi(x_0) = \mathcal{F}_{x_0}(\Phi)\ \ \ ,\forall \Phi \in \mathcal{C}_b(H)$. (14)

Then , we can verify that $\mathcal{F}_{x_0}$ is appositive functional from the $\mathcal{C}_b(H)$ *into* $\mathbb{R}$, and $\mathcal{F}_{x_0}$ is invariant for $\mathcal{P}_t$, given that $\mathcal{F}_{x_0}(\mathcal{P}_t\Phi) = \mathcal{F}_{x_0}(\Phi)\ \ \ ,\forall \Phi \in \mathcal{C}_b(H)$ *and* $t \geq 0$. (15)

Setting $\Phi = \mathcal{P}_s(\Phi),\ \Phi \in \mathcal{C}_b(H) \Rightarrow\ \lim_{t\to+\infty} \mathcal{P}_t\mathcal{P}_s\Phi(x_0) = \mathcal{F}_{x_0}(\mathcal{P}_s\Phi)$.

Again , $\lim_{t\to+\infty} \mathcal{P}_t\mathcal{P}_s\Phi(x_0) = \lim_{t\to+\infty} \mathcal{P}_{t+s}\Phi(x_0) = \mathcal{F}_{x_0}(\Phi)$.

(15) follows. Let $\mu T(E) = \frac{1}{T}\int_0^T \pi_t(x_0,E)dt,\ \ E \in \mathcal{B}(H), T > 0$. (16)

(16) is valid since the mapping $[0,+\infty) \to \mathbb{R},\ t \to \pi_t\big(x_{0,}E\big) = \mathcal{P}_t 1_E(x_0)$ *is Borel*

THEOREM 2:*Let* $\mathcal{P}_t$ be a Markov semigroup. Assume that for some $x_0 \in H$, *the set* $(\mu T)T > 0$, defined by (16) is tight. Then there is an invariant measure.

Proof : By the Prokhorov theorem, there exists a sequence $T_n \uparrow \infty$ and a probability measure $\mu \in \mathcal{P}(H)$*such that* $\lim_{n\to\infty} \int_H \Phi(x)\mu T_n(dx) = \int_H \Phi(x)\mu(dx)\ \ \forall \Phi \in\ \mathcal{C}_b(H)$, by Fubini theorem, we can recast it as $\lim_{n\to\infty} \frac{1}{T_n}\int_0^{T_n} \mathcal{P}_t\Phi(x_0)dt =\ \int_H \Phi(x)\mu(dx)\ \ \forall \Phi \in\ \mathcal{C}_b(H)$. (17)*Let* $\Phi = \mathcal{P}_s\Phi$, *for* $s \geq 0$, $\Phi \in\ \mathcal{C}_b(H)$since$\mathcal{P}_t$ is Feller.

Again , $\lim_{n\to\infty} \frac{1}{T_n}\int_0^{T_n} \mathcal{P}_{t+s}\,\Phi(x_0)dt = \int_H \mathcal{P}_s\Phi(d\mu)\forall \Phi \in\ \mathcal{C}_b(H)$. (18)

We need to establish that $\lim_{n\to\infty} \frac{1}{T_n}\int_0^{T_n} \mathcal{P}_{t+s}\,\Phi(x_0)dt = \int_H \Phi(d\mu)$is invariant. From (17),
$\lim_{n\to\infty} \frac{1}{T_n}\int_0^{T_n} \mathcal{P}_{t+s}\,\Phi(x_0)dt = \frac{1}{T_n}\int_0^{T_n+s} \mathcal{P}_t\,\Phi(x_0)dt$

$$= \frac{1}{T_n}\int_0^{T_n} \mathcal{P}_t\Phi(x_0)dt + \frac{1}{T_n}\int_{T_n}^{T_n+s} \mathcal{P}_t\Phi(x_0)dt - \frac{1}{T_n}\int_0^{s} \mathcal{P}_t\Phi(x_0)dt\ \ \Rightarrow \int_H \Phi(x)\mu(dx)\ as\, n \to \infty.$$

3 Existence Of An Invariant Measure

The existence and the uniqueness of an invariant measure is considered in this section.
Let $(\Omega, \mathcal{F}, \mathbb{P})$be a complete probability space, we consider a multidimensional diffusion process $X(t) = X_t$ which is a solution of the following stochastic differential problem

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t$$

$$X_0 = u_0 \in \mathbb{R}^d. \qquad (19)$$

Where $(W_t)_{t\in[0,T]}$ is the standard d-dimentional Brownian motion,$d \in N$, the drift coefficient $b: \mathbb{R}^d$ and $\sigma: \mathbb{R}^d \to \mathcal{L}(\mathbb{R}^m, \mathbb{R}^d)$, $m \in N$ are Lipschitz continuous function. The problem of (19) admits the following stochastic representation:
$S_t\, \varphi(x) = \mathbb{E}[\varphi(X(t,x)], x \in \mathbb{R}^d, \varphi \in \mathcal{C}_b(\mathbb{R}^d), where\ S_t, t \geq 0$, is the corresponding transition semi group, $\mathcal{C}_b(\mathbb{R}^d)$ denotes the space of all functions from $\mathbb{R}^d$into $\mathbb{R}$ that are uniformly continuousand bounded, $\mathbb{E}$ denotes the conditional expectation.
If $u_0: \mathbb{R}^d \to \mathbb{R}$ is a regular function, then the following function

$$u(t,x) = (S_t u_0)(x) = \mathbb{E}[u_0(X_t)]. \qquad (20)$$

Is the unique solution of the following problem:

$$\frac{\partial U}{\partial t} + \mathcal{L}u = 0\ in\ [0, +\infty\ ) \times \mathbb{R}^d\ , u(0,x) = u_0\ in\ \mathbb{R}^d. \qquad (21)$$

where $\mathcal{L}$is the linear , second-order uniformly elliptic operator associated with a diffusion process in the defined space.
The infinitesimal generator of the process (19) is given as

$$\mathcal{L} = -a(x): \mathcal{D}^2 - b(x)\nabla. \qquad (22)$$

Where the matrix $a(x) = (a_{ij}(x))$ is defined as follows:

$$a_{ij}\,(x) = \frac{1}{2}\sum_{\gamma=1}^{k} \sigma_{i\gamma}(x)\sigma_{j\gamma}(x) \quad and \quad a(x): \mathcal{D}^2 = trace[a\mathcal{D}^2] = \sum_{i,j=1}^{d} a_{ij}\, \partial_{ij} \qquad (23)$$

∇ *and* $\mathcal{D}^2$denote the gradient and the Hessian operators with respect to the spatial variable x respectively.

Furthermore, state below the proposition of the auxiliary mapping of Lyapunov function to establish the existence of invariant measure.

PROPOSITION 2: *Let* $V: H \to [0, +\infty]$ be a Borel function whose level sets

$K_a = \{x \in H: V(x) \leq a\}, a > 0$are compact for any $a > 0$. Assume that $\exists\ x_0 \in \mathbb{R}^n$ *and* $\mathcal{C}(x_0) > 0$

Such that $[V(X(t, x_0))] \leq \mathcal{C}(x_0)\ , \forall\ t \geq 0$. (24)
Then Λ is nonempty.

If in addition $\exists\ \mathcal{C} > 0$ such that $\mathbb{E}\big[V\big(X(t,x)\big)\big] \leq \mathcal{C}\ \forall\ t \geq 0, x \in H$ (25)

Then Λ is tight and $\int_{R^n} V(x)\mu(dx) \leq \mathcal{C}\, \forall\ \ \mu \in \Lambda$. (26)

Example: (Heat Equation)

Let $\Lambda \subset \mathbb{R}^d$be a bounded open set with a $\mathcal{C}^2$ boundary $\partial\Lambda$. Consider $E = \mathcal{L}^2(\Lambda)$, and

$Let\ F: E \to E\ and\ G: E \to \mathcal{L}(H, E)$beLipschitz maps with Lipschitz constants $L_F\ and\ L_G$ respectively.$Let\ H$ be a separable real Hilbert space on which we consider a stationary independent increments process $\mathcal{Z} = (\mathcal{Z}(t))_{t\geq 0}$ defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with filtration $(\mathcal{F}_t)_{t\geq 0}$.

Assume $(0) = 0\ \ and\ sup_{0\leq t\leq T}\ \mathbb{E}\|\mathcal{Z}(t)\|^2 < \infty \quad \forall\, T > 0.$ (27)

Consider a nonlinear stochastic heat equation:

$\frac{\partial u}{\partial t}(t,x) = \Delta_x u(t,x) + F\big(u(t,\cdot)\big)(x) + (G\big(u(t,\cdot)\big)dZ(t)(x)\ \ for\ \ x \in \Lambda\,, t \geq 0$, with initial and boundary conditions.

$$u(0,x) = u_0(x)\,, \qquad x \in \Lambda$$

$u(t,x) = 0,\ x \in \partial\Lambda, where\ u_0$is an E- valued $\mathcal{F}_0$ - measurable random variable.

The operator $A = \Delta_x\ defined\ on\ D(A) = \{u \in H^2(\Lambda): u = 0\ on\ \partial\Lambda\} \subset E$ generates a strongly continuous semigroup $(S(t))_{t\geq 0}\ on\ E\ such\ \ that\ \|S(t)\| \leq \exp(-\alpha t)\ \forall\, t \geq 0\ \ where\ -\infty < 0$ is the eigenvalue of A.

There exists a unique invariant measure μ for (20) on $with\ \ \int_E \|x\|^2 d\mu < \infty$, whenever $K = 6(L_F^2/\alpha + L_G^2\sigma) < \alpha,\ \ where\ \sigma\ is\ given\ by\ (E\|Z(1) - EZ(1)\|^2)^{1/2}$, $L_G = 0$ for additive noise.

Conclusively , by $Lipschitz\ condition,\ \ if\ F$ is given by $F(u) = x \to f\big(u(x)\big), u \in L^2$ (Λ) for some continuous function $f: \mathbb{R} \to \mathbb{R}$, $then\ \ L_F \leq L_f$, where L_f is the Lipschitz constant of f.

4. Conclusion :We have examined the invariant measures for stochastic evolution equations in Hilbert space and derive a sufficient condition for the existence of an invariant measure in a global case for a more corrupted noise than Wiener processes. We conclude the work by the problem of Heat equation. The conditions can be extended in the solution to the problems of classical linear case by heat equation by relaxing the conditions, that is $F = 0\ \ and\ L_G = 0$ (see [16])

References

[1] Chojnowska- Michalik, A. and Goldys, B., (1995).*Existence, uniqueness and invariant measures for stochastic semilinear equations in Hilbert spaces*, Probab. Th. Rel. Fields *102*, 331–356.

[2] Da Prato, G., Elworthy, K.D. and Zabczyk, J. (1995), *Strong Feller property for stochastic semilinear equations*, Stoch. Anal. Appl. *13*, 35–45.

[3] Da Prato, G., Gatarek, D. and Zabczyk, J. (1992).*Invariant measures for semilinear stochastic equations*, Stoch. Anal. Appl. *10*, 387–408.

[4] Da Prato, G. and Zabczyk, J., (1996).*Ergodicity for Infinite Dimensional Systems*, (Cambridge University Press, Cambridge.

[5] Es- Sarhir, A. and Farkas, B. (2007).*Invariant measures and regularity properties of perturbed Ornstein-Uhlenbecksemigroups*, J. Diff. Equ. *233*, 87–104.

[6] Es- Sarhir, A. and Stannat, W. (2008).*Invariant measures for semilinear SPDE's with local Lipschitz drift coefficients and applications*, J. Evol. Equ. *8*, 129–154.

[7] Ichikawa, A. (1984).*Semilinear stochastic evolution equations: Boundedness, stability and invariant measures*, Stochastics*12*, 1–39.

[8] Leha, G. and Ritter, G. (1994).*Lyapunov-type conditions for stationary distributions of diffusion processes on Hilbert spaces*, Stoch. Stoch. Rep. *48*, 195–225.

[9] Maslowski, B., *Strong Feller property for semilinear stochastic evolution equations and applications*, Stochastic systems and optimization (Warsaw, 1988), pp. 210–224, Lecture Notes in Control Inform. Sci. 136, Springer-Verlag, Berlin, 1989.

[10] Maslowski, B. (1989).*Uniqueness and stability of invariant measures for stochastic differential equations in Hilbert spaces*, Stoch. Stoch. Rep. *28*, 85–114.

[11] Maslowski, B. and Seidler, J. (1998).*Invariant measures for nonlinear SPDE's: uniqueness and stability*, Arch. Math. *34*, 153–172.

[12] Peszat, S. and Zabczyk, J. (1995).*Strong Feller property and irreducibility for diffusions on Hilbert spaces*, Ann. Probab. *23*, 157–172.

[13] J. Jacod, Calculstochastique et probl_emes de martingales, Lecture Notes in Mathematics 714, Springer-Verlag, Berlin, 1979.

[14] O.Ijioma. (2021). *"Invariant Measure For A Class Of Stochastic Evolution Equations Driven By Levy Process". IOSR Journal of Mathematics (IOSR-JM, 17(4), pp47-52.*

[15] O. van Gaans and S. VerduynLunel, (2004). Long term behavior of dichotomous stochastic differential equations in Hilbert spaces, Commun. Contemp. Math.6, No. 3, 349-376.

[16] A. Chojnowska-Michalik, (1997).“Stationary solutions for heat equation perturbed bygeneral additive noise”, *J. Appl. Anal. 3, no. 1, 129-136.*

[17] M.C. Delfour,(1980). “The largest class of hereditary systems defining a C0 semigroup on the product space”, *Can. J. Math. 32, no. 4, 969-978.*

[18] A.A. Guschchin and U. Kuchler, (2000) “On stationary solutions of delay differentialequations driven by a Lévy process”, *Stochastic Process. Appl. 88, 195-211*.

[19] S.-E.A. Mohammed and M.K.R Scheutzow, (1996). “Lyapunov exponents of linear stochastic functional differential equations driven by semi martingales”, *Ann. Inst. Henri Poincare 32, no.1, 69-105.*

[20] O. van Gaans, (2005). “A series approach to stochastic differential equations with infinite-dimensional noise”, *Integral Equations Operator Theory 51, no. 3),435-458.*

[21] O. van Gaans and S. VerduynLunel, (2004). “Long term behavior of dichotomous Stochastic differential equations in Hilbert spaces”, *Commun. Contemp. Math. 6, No. 3, 349-376.*